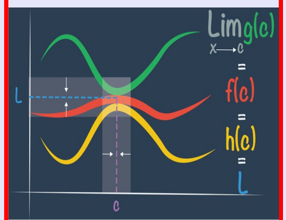
Math 261

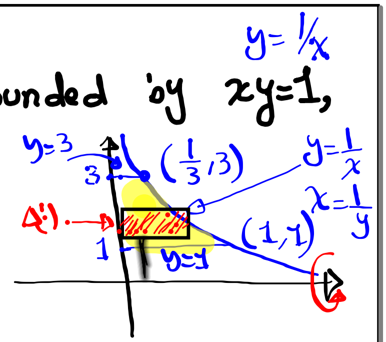
Spring 2021

Lecture 52



Consider the region bounded by $xy=1$, ($y=1/x$) $x=0$, (y-axis) $y=1$, and $y=3$.

1) Draw & clearly label.



2) Rotate about x-axis, find the volume.

Ref. Rect. ∥ A.O.R.

Shells

$$V=\int_1^3 2\pi y\left(\frac{1}{y}-0\right)dy = 2\pi\int_1^3 1\,dy = \boxed{}$$

3) Rotate about y-axis, find the volume.

Ref. Rect. ⊥ A.O.R.
Region is attached to A.O.R. } Disk

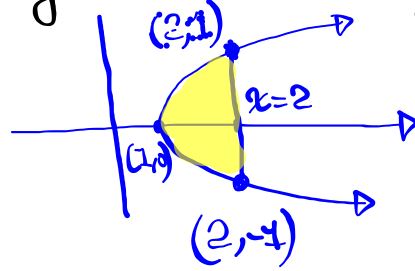
$$V=\int_1^3 \pi\left[\frac{1}{y}\right]^2 dy = \pi\int_1^3 y^{-2}\,dy = \boxed{}$$

1) Draw the region enclosed by $x-y^2=1$ and $x=2$.

$x=y^2+1$

$2=y^2+1$

$1=y^2 \rightarrow y=\pm 1$



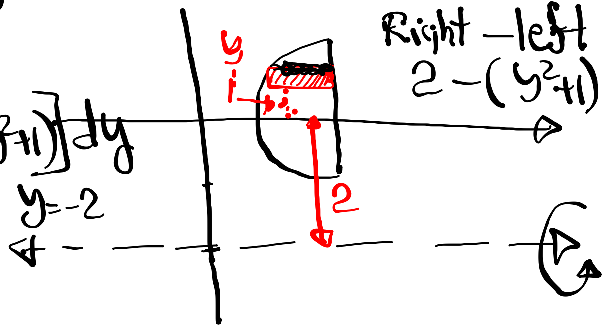
2) Rotate about $y=-2$, find its volume.

Shells

$$V=\int_{-1}^{1} 2\pi(y+2)\cdot\left[2-(y^2+1)\right]dy$$

Right − left

$2-(y^2+1)$



Find the volume generated by rotating the region bounded by $y=x$ and $y=\dfrac{2x}{1+x^3}$ about $x=-1$.

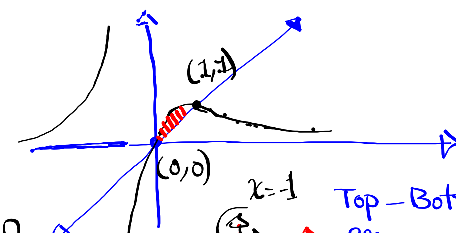
$$\frac{2x}{1+x^3}=x$$

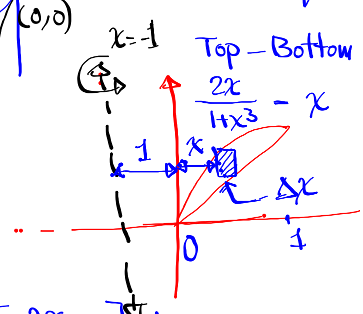
$$x(1+x^3)=2x$$

$$x(1+x^3-2)=0$$

$$x(x^3-1)=0$$

$\boxed{x=0}$ $\quad x^3-1=0 \rightarrow \boxed{x=1}$



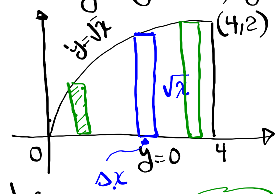


$$V=\int_0^1 2\pi(x+1)\cdot\left[\frac{2x}{1+x^3}-x\right]dx$$

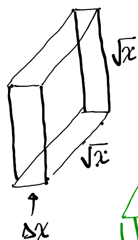
Volume of Solids (No rotation)

Draw a region enclosed by $y=\sqrt{x}$, $y=0$, and $x=4$.

Draw a Vertical Ref. Rectangle



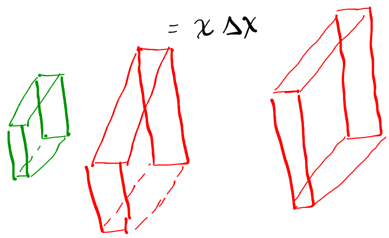
Make a box with its bas as my ref. Rectangle, Make this box as square base



Volume = LWH

$= \sqrt{x}\sqrt{x} \cdot \Delta x$

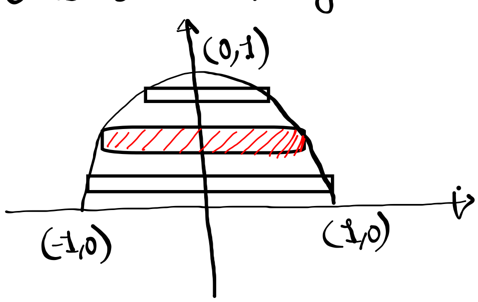
$= x\,\Delta x$

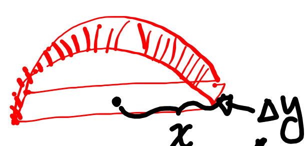


$$V=\int_0^4 (\sqrt{x})^2\,dx = \frac{x^2}{2}\Big|_0^4 = \boxed{8}$$

The base of a Solid is the region enclosed by $y=1-x^2$ and x-axis. $\rightarrow x^2=1-y$

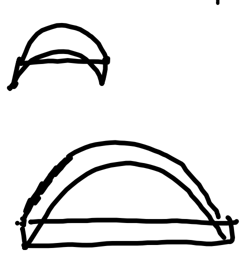
Cross-Sections $\perp$ y-axis and are Semi-Circles.





$$\frac{\pi x^2 h}{2}$$

$$\frac{\pi(1-y)\Delta y}{2}$$



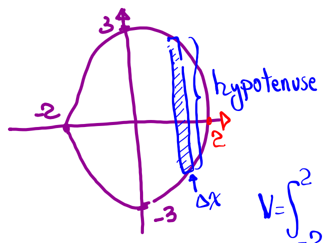
$$V=\int_0^1 \frac{\pi(1-y)}{2}\,dy$$

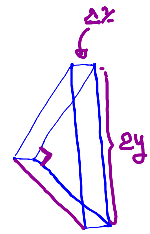
$y^2 = \frac{36-9x^2}{4}$

The base of Solid is the enclosed region by $9x^2+4y^2=36. \to \frac{x^2}{4}+\frac{y^2}{9}=1$

Cross-sections $\perp$ x-axis.

Cross-Section are isosceles right Triangle with hypotenuse in the base.

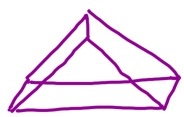




$$V=\int_{-2}^{2} y^2 \, dx$$

$$= 2\int_{0}^{2} \frac{36-9x^2}{4}\, dx$$

$$= \boxed{}$$



$A=\frac{1}{2}bh$

$=\frac{1}{2}\cdot 2y \cdot y$

$A=y^2$